7.5 – Complex Numbers

Daily Objectives:

1. Define complex numbers as numbers of the form $a+bi$, where $i^2=-1$.
2. Identify and find the conjugate of a complex number.
3. Find nonreal solutions as conjugate pairs.
4. Explore arithmetic computations with complex numbers.

The imaginary unit i *is defined by:*

$$i=\sqrt{-1}$$

$$i^2=\left(\sqrt{-1}\right)^2=-1$$

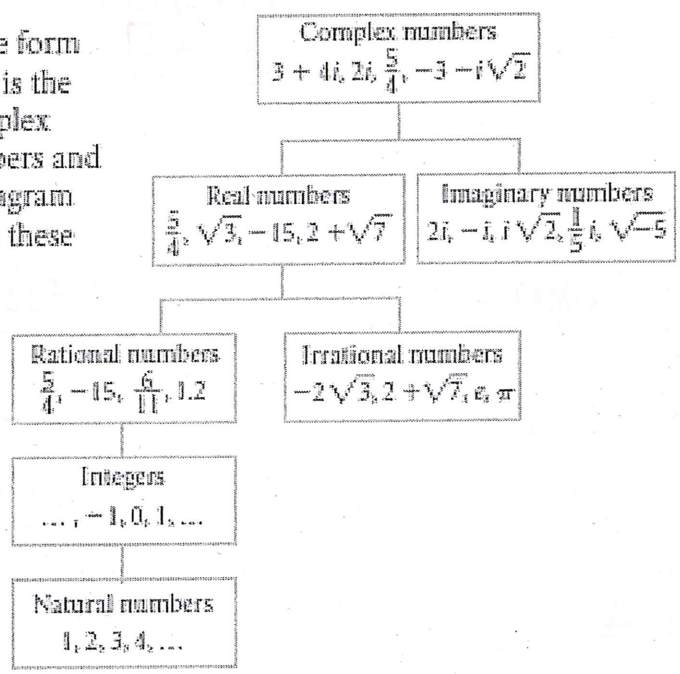
Using this definition find each of the following:

$i^0 = 1$
$i^1 = i$
$i^2 = -1$
$i^3 = i \cdot i^2 = i \cdot -1 = -i$
$i^4 = i^2 \cdot i^2 = -1 \cdot -1 = 1$
$i^5 = i^4 \cdot i = i$
$i^6 = i^4 \cdot i^2 = -1$

Complex Numbers

A **complex number** is a number in the form $a+bi$, where a and b are real numbers and $i=\sqrt{-1}$.

For any complex number in the form $a+bi$, a is the real part and bi is the imaginary part. The set of complex numbers contains all real numbers and all imaginary numbers. This diagram shows the relationship between these numbers and some other sets you may be familiar with, as well as examples of numbers within each set.



1 Solve $x^2+3=0$.

Example 1: Graph the equation $x^2+3=0$.

a. How many x-intercepts do you see? None

When a quadratic does not have *real* intersections with the x-axis, you will have imaginary roots.

Imaginary roots come in pairs. If you have one imaginary root, you will have another – called its conjugate root.

b. Solve for x: $x^2+3=0$

$-3 \quad -3$

$x^2=-3$

$x=\pm\sqrt{-3}$

$x=\pm\sqrt{-1\cdot 3}$

$x=\pm\sqrt{-1}\sqrt{3}$

$\boxed{x=\pm i\sqrt{3}}$

Example 2: Solve for x: $x^2+25=0$

$-25 \quad -25$

$\sqrt{x^2}=\sqrt{-25}$

$x=\pm\sqrt{-25}$

$x=\pm\sqrt{25}\sqrt{-1}$

$\boxed{x=\pm 5i}$

Example 3: Solve for x:

a. $x^2-2x+6=0$

$x=\dfrac{2\pm\sqrt{(-2)^2-4(1)(6)}}{2(1)}$

$x=\dfrac{2\pm\sqrt{4-24}}{2}$

$x=\dfrac{2\pm\sqrt{-20}}{2}$

$x=\dfrac{2\pm i\sqrt{20}}{2}$

b. $3x^2+x+2=0$

$\dfrac{-1\pm\sqrt{1^2-4(3)(2)}}{2(3)}$

$\dfrac{-1\pm\sqrt{1-24}}{6}$

$\dfrac{-1\pm\sqrt{-23}}{6}$

$\dfrac{-1\pm\sqrt{-1}\sqrt{23}}{6}$

$x=\dfrac{-1\pm i\sqrt{23}}{6}$

Investigation • Complex Arithmetic

When computing with complex numbers, there are conventional rules similar to those you use when working with real numbers. In this investigation you will discover these rules. You may use your calculator to check your work or to explore other examples. [▶ See Calculator Note 7E to learn how to enter complex numbers into your calculator.◀]

Part 1: Addition and Subtraction

Addition and subtraction of complex numbers is similar to combining like terms. Use your calculator to add these complex numbers. Make a conjecture about how to add complex numbers without a calculator.

a. $(2 - 4i) + (3 + 5i)$

$5 + i$

b. $(7 + 2i) + (-2 + i)$

$5 + 3i$

c. $(2 - 4i) - (3 + 5i)$

$2 - 4i - 3 - 5i$

$-1 - 9i$

d. $(4 - 4i) - (1 - 3i)$

$4 - 4i - 1 + 3i$

$3 - i$

Part 2: Multiplication

Use your knowledge of multiplying binomials to multiply these complex numbers. Express your products in the form $a + bi$. Recall that $i^2 = -1$.

a. $(2 - 4i)(3 + 5i)$

$6 + 10i - 12i - 20i^2$

$6 - 2i - 20(-1)$

$6 - 2i + 20$

$26 - 2i$

b. $(7 + 2i)(-2 + i)$

$-14 + 7i - 4i + 2i^2$

$-14 + 3i + 2(-1)$

$14 + 3i - 2$

$12 + 3i$

c. $(2 - 4i)^2$

$(2 - 4i)(2 - 4i)$

$4 - 8i - 8i + 16i^2$

$4 - 16i + 16(-1)$

$4 - 16i - 16$

$-12 - 16i$

d. $(4 - 4i)(1 - 3i)$

$4 - 12i - 4i + 12i^2$

$4 - 16i + 12(-1)$

$4 - 16i - 12$

$-8 - 16i$

Part 3: The Complex Conjugates

Recall that every complex number $a + bi$ has a complex conjugate, $a - bi$. Complex conjugates have some special properties and uses.

Each expression below shows either the sum or product of a complex number and its conjugate. Simplify these expressions into the form $a + bi$, and generalize what happens.

a. $(2 - 4i) + (2 + 4i)$

4

b. $(7 + 2i) + (7 - 2i)$

14

c. $(2 - 4i)(2 + 4i)$

$4 + 8i - 8i - 16i^2$

$4 - 16(-1)$

$4 + 16$

20

d. $(-4 + 4i)(-4 - 4i)$

$16 + 16i - 16i - 16i^2$

$16 - 16(-1)$

$16 + 16$

32

Example 4: If a quadratic has a root of $5i$, what is its other root?

$-5i$

$x^2 + 25 = 0$

$x^2 = -25$

Example 5: A quartic is a polynomial with degree 4. If a quartic, has roots $-9i$ and $3 + i$, what are the other two roots?

$+9i$ $3 - i$